**Chapter 3**

**Background Theory and Mathematical Modelling**

**3.1 The General Energy Equation**

Guo’s approach to bottom-hole pressure estimations for producing oil wells involves the solution of a general energy equation-derived model using the Newton-Raphson numerical procedure for the hydraulic and friction pressure at depth; the friction gradient term also being neglected.

This general energy equation expresses and energy equilibrium between two points in a fluid flow system. It is also another statement of the general principle of conservation of energy.

It states that the sum of the energy of the fluid entering a section of a pipe (section A) and any other additional work done on the fluid between this section and the next (section B) minus the energy losses by the system between sections A and B equals the energy of the fluid leaving section B.

The general energy equation for a fluid flow system used in the Guo model is expressed as:

*U*1+ + 1.00)

Rewriting 1.00) in units of ft-lbf, the equation is then expressed as;

1.01)

But, thus 1.01) is then simplified as;

1.02)

Transferring all the terms on the left hand side to the right hand side and then equating to zero, the equation becomes;

Δ Δ Δ + Δ 1.03)

Where;

Δ is the change in the internal energy of the fluid from the pipe inlet to the outlet.

Δ is the change in the kinetic energy of the fluid from the pipe inlet to the outlet.

Δ is the change in potential energy of the fluid from pipe inlet to outlet.

Δ is the energy of compression (or expansion) of the fluid.

*q* is the heat transferred to or away from the fluid.

*w* is the work done on or by the gas.

The internal energy-entropy relation of a thermodynamic system from the first law of thermodynamics is then incorporated into the Guo model. The mathematical expression for this internal energy-entropy relation is given as;

I)

Integrating both sides and considering an infinitesimal change in internal energy U, the relation equation is then expressed as;

Δ 1.04)

Where;

is the effect of heat on the system.

is the effect of compression on the system.

A symbolic representation of the entropy is then given by;

= II)

Where;

is the work lost due to external factors e.g. spillage loss of the fluid.

Substituting II) into 1.04), an expression for the internal energy is derived and is of the form;

Δ 1.05)

If the term Δ(*PV*) is also expressed as a complete differential, it will take the form;

Δ III)

Substituting III) and 1.05) into 1.03), another expression is obtained given as;

Further evaluation of the expression gives;

+ =0 1.06)

For an lbm unit mass of fluid, 1.06) is expressed as;

+ =0 IV)

Rewriting IV) in a differential form, the expression below is obtained;

1.07)

Differentiating 1.07) with respect to Z and then multiplying by density ρ, we obtain;

V)

This gives;

VI)

For a unit mass, we recall that ρV=1; which implies that VI) becomes;

VII)

Making necessary adjustments, the following expression is obtained;

1.08)

Defining the fiction gradient and considering the angle of multiphase flow,

= (Srichai S., 2010) VIII)

1.09)

has no mathematical interpretation hence it is equal to zero.

IX)

Where;

is the total pressure gradient.

is the acceleration/kinetic gradient.

is the elevation/potential gradient.

is the friction loss gradient.

The negative sign shows the decrease in pressure with an increase in height.

Note that for a vertical multiphase flow, θ=.

1.10)

Introducing specific weight,

This implies that;

Using Newton’s Second law,

In lb-ft units, k=

It can then be evaluated that;

, *W*=

But

This implies that;

In most cases,,

This shows that

This implies that;

1.11)

Equation 1.11) is the general energy equation for characterizing pressure drop associated with fluid flow in a vertical pipe.

The general energy equation for characterizing pressure drop associated with fluid flow in a vertical pipe according to Guo’s base model is given by:

1.12)

Guo’s approach involves the solution of 1.12) using Newton-Raphson numerical procedure for the hydraulic and friction pressure at depth.

**3.2 The Extension Model Development**

This revised model is an extension of Guo’s four phase model for multiphase fluid flow. A few modifications are made to this model by:

1. Addition of a friction gradient component that was neglected in the base model.
2. The use of flowing bottom-hole temperature rather than average tubing temperature in the estimation.
3. The use of definite integrals with limits between wellhead pressure and bottom-hole pressure in the estimations.

**3.2.1 Model Assumptions**

1. The acceleration pressure gradient term in the energy term is negligible as a result of the small changes in the multiphase fluid velocity over incremental distance.
2. The fluid flow is steady and isothermal and hence no work is done by the gas during its flow.
3. Fluid flow velocity is based on liquid and gas flow rates only, with the volumetric flow rate of solid being negligible compared to those of the liquid and gas.
4. Liquid holdup effects are neglected based on small diameter tubing’s with average high mixture flow rates.

Applying this extension model’s assumptions to 1.11) and neglecting the acceleration pressure gradient, we then have that;

1.13)

Where is the internal diameter of the pipe.

All the terms are modified to account for the three phase fluid mixture flow. The mathematical expressions for the specific weight and fluid flow velocity are the derived below.

The specific weight of the mixture is expressed as;

1.14)

Where;

W is the rate flow mixture in lb/sec

q is the volumetric flow rate of substance (solid or fluid)

**For a solid**, the volumetric flow rate of solid is expressed as

1.15)

The weight flow rate () of a substance is given by the product of its density and the volumetric flow rate of the substance. Thus, we have that;

lb/sec 1.16)

**For a liquid**, the volumetric flow rate can be expressed as;

1.17)

The weight flow rate is expressed as;

lb. /sec

lb /sec 1.18)

**For a gas**, the volumetric flow rate is derived and evaluated with the gas flow at standard conditions based on the real gas law.

Where;

is the atmospheric pressure in psia.

is the surface temperature in oR.

At standard conditions of gas flow, we express as;

1.19)

The weight flow rate *Wg* in this case depends only on the volumetric gas flow rate at surface conditions, and the specific gravity.

lb. /sec 1.20)

Substituting 1.15) trough to 1.20) into 1.14), we then get the expression below;

Simplifying the above expression further, we then get;

1.21)

This can be rewritten as a)

Where;

The fluid mixture flow velocity can be derived based on oil and gas flow rates and the cross sectional area of the pipe. The mixture’s flow velocity and the cross sectional area is connected by the equation;

Making necessary conversions, we have that;

ft/sec 1.22)

This can be rewritten as;

b)

Where;

Substituting a) and b) into 1.13), we get the expression for the pressure differential;

1.23)

Where;

*f is* the friction factor of the pipe and is given by the Nikuradse’s friction factor correlation (1933)

The pressure differential is then integrated within the limits of the wellhead pressure and the flowing bottom-hole pressure.

1.24)

1.25)

The pressure differential is then split into three independent components as shown below;

1.26)

This implies that;

All components are then integrated by parts as illustrated below;



Making appropriate conversions of pressure units to psia since is already in psia, we have;

1.27)



Since is already in psia, we then convert to units of psia, which gives us;

1.28)

Converting to units of psia, we then have;

1.29)

The equations 1.26), 1.27) and 1.28) above are then solved using Newton-Raphson’s numerical method of solving polynomials to obtain P1,P2 and P3 respectively.

Newton-Raphson’s method of tangents numerical algorithm is used to obtain the real square roots of third or higher degree algebraic equations.

After determining the roots of the equations above, the flowing bottom-hole pressure is given by

Methods of Solution

1. **The Ideal gas case solution**

* The coefficients **a,b,c,d** and **e** in the model equations are first evaluated with data acquired from the field. For the ideal case, a compressibility factor (Z) of 1 is used.
* The Newton-Raphson’s numerical algorithm is then used to calculate pressures P1, P2 and P3 from equations 1.27), 1.28) and 1.29) respectively. The estimated bottom-hole pressure for the vertical oil well is the sum of pressure components P1, P2 and P3.

1. **The Real gas case solution**

* Since the compressibility factor Z is dependent on the pressure being estimated, an iterative procedure is used. The bottom-hole pressure is first estimated using the ideal gas case conditions. This becomes the initial condition for the iterative procedure.
* The estimated bottom-hole pressure in conjunction with the pseudo critical temperature and pressure (Ppc and Tpc) are then used to determine the pseudo reduced pressure and temperature (Ppr and Tpr).
* The compressibility factor is then traced using Ppr and Tpr on a Standing and Katz chart. This compressibility factor is then substituted back into 1.27), 1.28) and 1.29) to determine the flowing bottom-hole pressure for the real gas case. Much iteration is carried out for a higher degree of accuracy.
* The final bottom-hole pressure for the real case behavior of gases is derived when the compressibility factor Z converges.